ON CLOSED-FORM CALCULATION OF CVAR

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ABSTRACT. Although Conditional Value-at-Risk has significant advantages over traditional risk measures such as Value-at-Risk, it has not been adopted by practitioners as quickly as expected. One of the reasons slowing down its progress has been the lack of simple tools for its computation. In this paper we consider calculating CVaR when the underlying asset is modelled using a diffusion process with a linear drift and prespecified marginal density. The results are summarized in two closed-form formulas which can be effortlessly applied by risk managers to calculate CVaR for a number of commonly used probability distributions. Example of calculations is included.
KEY WORDS: conditional value-at-risk, coherence, risk measure, expected shortfall

1. INTRODUCTION

The problem of finding risk measures that appropriately penalize the tails has received considerable attention in the last few years. Although Value-at-Risk (VaR) is still widely used for measuring extreme events and integrating disparate sources of risk, its limitations are increasingly recognized (Szegö (2002), Daníelsson (2002)). The main caveat of VaR is that it is not a convex functional when non-elliptical distributions are considered, which makes it inappropriate for portfolio-optimization problems. Further the lack of sub-additivity implies that portfolio diversification may lead to an increase in risk and prevent to add up the VaR of different risk sources. Thus VaR is not coherent in Artzner et al. (1999) sense and regulatory agencies should be careful about insisting its use. As discussed by Rockafellar and Uryasev (2002) a serious shortcoming of VaR is that it merely provides a lowest bound for losses without being able to distinguish between their degrees. Therefore VaR has a bias toward optimism instead of conservatism that ought to prevail in risk management.

In response an alternative risk measure, Conditional Value-at-Risk (CVaR), has been proposed to replace VaR. The definition of CVaR is relatively intuitive: for general distributions CVaR is defined as the weighted average of VaR and the

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