Conditional value-at-risk estimation using non-integer values of degrees of freedom in Student's *t*-distribution

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This paper provides an analytical formula for CVAR calculated using *t*-distributions with non-integer degrees of freedom. We generalize standard formulas, calculated on the assumption of normal log-returns without compromising on the difficulty of the calculation procedure involved. We also extend the results of Heikkinen and Kanto (2002) to show the impact of kurtosis on values of CVAR. The results are summarized in a closed-form formula that can, with little effort, be used by risk managers in the evaluation risk exposures for a family of heavy-tailed distributions.

1 Introduction

Risk managers and regulators need measures for the evaluation of risk. There are two common measures of risk: value-at-risk (VAR) and conditional value-at-risk (CVAR) (Jorion, 2000; Artzner *et al*, 1999). The former is the lower bound that is reached with given probability, usually 95%, 97.5%, 99% or 99.9%. The latter gives the expected loss assuming that the lower bound is reached.

The simplest way to proceed is to assume that returns are normally distributed. However, in practice this assumption seldom holds because the tails are heavier than in the normal case. One possible alternative is Student's *t*-distribution (Student, 1908), which has slightly heavy tails. In his seminal article, Student considered distributions with integer degrees of freedom, but mathematically this is not necessary. In this paper, therefore, we allow degrees of freedom to be non-integers. A nice property of this class of distributions is that kurtosis and degrees of freedom have a simple relationship. Thus, degrees of freedom can easily be estimated using the method of moments. In practice, the kurtosis is often larger than six, leading to non-integer degrees of freedom of between four and five.

The critical values of Student's *t*-distribution with integer values are often reported in standard textbooks. Recently, Heikkinen and Kanto (2002) reported them with several non-integer values. In this paper we present a compact formula for CVAR with non-integer degrees of freedom. It shows that if the data are

The detailed comments of the Editor-in-Chief are gratefully acknowledged.