

COHERENT MEASURES OF RISK

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In this paper we study both market risks and nonmarket risks, without complete markets assumption, and discuss methods of measurement of these risks. We present and justify a set of four desirable properties for measures of risk, and call the measures satisfying these properties “coherent.” We examine the measures of risk provided and the related actions required by SPAN, by the SEC/NASD rules, and by quantile-based methods. We demonstrate the universality of scenario-based methods for providing coherent measures. We offer suggestions concerning the SEC method. We also suggest a method to repair the failure of subadditivity of quantile-based methods.

KEY WORDS: aggregation of risks, butterfly, capital requirement, coherent risk measure, concentration of risks, currency risk, decentralization, extremal events risk, insurance risk, margin requirement, market risk, mean excess function, measure of risk, model risk, net worth, quantile, risk-based capital, scenario, shortfall, subadditivity, tail value at risk, value at risk

1. INTRODUCTION

We provide in this paper a *definition* of risks (market risks as well as nonmarket risks) and present and justify a unified framework for the analysis, construction, and implementation of *measures* of risk. We do not assume completeness of markets. These measures of risk can be used as (extra) capital requirements to regulate the risk assumed by market participants, traders, and insurance underwriters, as well as to allocate existing capital.

Initial manuscript received February 1996; final revision received September 1998.

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The authors acknowledge financial support from the Société Générale for this work. The views expressed are those of the authors. For useful discussions on a related paper, we thank participants of the following meetings: Boston University Mathematical Finance Day, March 31, 1996, University of Waterloo Insurance and Finance Conference, May 24, 1996, StudienZentrum Gerzensee Symposium in Financial Markets, July 15–26, 1996, Latsis Symposium, ETH Zürich, September 24–25, 1996, Universität Karlsruhe Geld, Finanzwirtschaft, Banken, Versicherungen Tagung, December 11–13, 1996, Aarhus University Workshop, February 25–March 1, 1997. We also thank D. Madan, discussant at the French Finance Association International Meeting, Geneva, June 1996, F. Diebold, discussant at the Federal Reserve Bank of Atlanta 1997 Financial Markets Conference, R. Bliss, P. Boyle, V. Brousseau, P. Embrechts, A. Hoffman, W. Neufeind, C. Petitmengin, P. Poncet, J. Renegar, and E. Shiu, as well as a referee of an earlier version of this paper.