

# Different Approaches to Risk Estimation in Portfolio Theory

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Almost forty years ago, Sharpe [1966] developed a measure then called reward-to-variability for evaluating and predicting the performance of mutual fund managers. Under the name of the Sharpe ratio, it has become one of the most popular indexes in practical application.

Although the Sharpe ratio is fully compatible with normally distributed returns (or, in general, with elliptical returns), it will lead to incorrect investment decisions when returns present kurtosis or skewness (see, among others, Leland [1999] and Bernardo and Ledoit [2000]). Specifically, returns on assets exhibit heavy tails, and many researchers recognize the limitations of this performance measure (see Ortobelli et al. [2003]).

Several alternatives to the Sharpe ratio for optimal portfolio selection have been proposed such as the min-max ratio, the stable ratio, the mean absolute deviation ratio, the Farinelli-Tibiletti ratio, and the Sortino-Satchell ratio; see Young [1998], Ortobelli et al. [2003a, 2003b], Farinelli and Tibiletti [2003a, 2003b], Sharpe [1994], Dowd [2001], Sortino [2000], Pedersen and Satchell [2002], Szegö [2004], and Uryasev [2000]. All these performance measures are theoretically valid and lead to different optimal solutions, so it is unclear how an investor should decide on a criterion.

Our goal is to find a criterion whose application would lead to correct investment decisions in the case of heavy-tail distributed returns. We use an ex post analysis considering recent historical data. In our study period, stock returns were volatile and far from normally (Gaus-