# Backtesting Parametric Value-at-Risk With Estimation Risk

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One of the implications of the creation of the Basel Committee on Banking Supervision was the implementation of Value-at-Risk (VaR) as the standard tool for measuring market risk. Since then, the capital requirements of commercial banks with trading activities are based on VaR estimates. Therefore, appropriately constructed tests for assessing the out-of-sample forecast accuracy of the VaR model (backtesting procedures) have become of crucial practical importance. In this article we show that the use of the standard unconditional and independence backtesting procedures to assess VaR models in out-of-sample composite environments can be misleading. These tests do not consider the impact of estimation risk, and therefore, may use wrong critical values to assess market risk. The purpose of this article is to quantify such estimation risk in a very general class of dynamic parametric VaR models and to correct standard backtesting procedures to provide valid inference in out-of-sample analyses. A Monte Carlo study illustrates our theoretical findings in finite-samples and shows that our corrected unconditional test can provide more accurately sized and more powerful tests than the uncorrected one. Finally, an application to the S&P 500 Index shows the importance of this correction and its impact on capital requirements as imposed by the Basel Accord.

KEY WORDS: Backtesting; Basel Accord; Conditional quantile; Estimation risk; Fixed, rolling, and recursive forecasting scheme; Forecast evaluation; Risk management; Value-at-Risk.

## 1. INTRODUCTION

In the aftermath of a series of bank failures during the seventies a group of 10 countries (G-10) decided to create a committee to set up a regulatory framework to be observed by internationally active banks operating in these member countries. This committee, coined as the Basel Committee on Banking Supervision (BCBS), was intended to prevent financial institutions, in particular banks, from operating without effective supervision.

The subsequent documents derived from this commitment focused on the imposition of capital requirements for internationally active banks intending to act as provisions for losses from adverse market fluctuations, concentration of risks, or simply bad management of institutions. The risk measure agreed to determine the amount of capital on hold was the Value-at-Risk (VaR). In financial terms, this is the maximum loss on a trading portfolio for a period of time given a confidence level. In statistical terms, VaR is a quantile of the conditional distribution of returns on the portfolio given agent's information set. More formally, denote the real-valued time series of portfolio returns or profit and losses (P&L) account by $Y_t$, and assume that at time $t-1$ the agent's information set is given by $W_{t-1}$, which may contain past values of $Y_t$ and other relevant economic and financial variables, i.e., $W_{t-1} = (Y_{t-1}, Z'_{t-1}, Y_{t-2}, Z'_{t-2}, \ldots)'$. Henceforth, $A'$ denotes the transpose matrix of $A$. Let $\mathcal{F}_{t-1}$ be the $\sigma$-algebra generated by $W_{t-1}$. Assuming that the conditional distribution of $Y_t$ given $W_{t-1}$ is continuous, we define the $\alpha$th conditional VaR of $Y_t$ given $W_{t-1}$ as the $\mathcal{F}_{t-1}$-measurable function $q_\alpha(W_{t-1})$ satisfying the equation

$$P(Y_t \le q_\alpha(W_{t-1}) \mid W_{t-1}) = \alpha \quad \text{almost surely (a.s.)}, \alpha \in (0,1)\ \forall t \in \mathbb{Z}. \quad (1)$$

In *parametric* VaR inference, one assumes the existence of a parametric family of functions $\mathcal{M} = \{m_\alpha(\cdot,\theta) : \theta \in \Theta \subset \mathbb{R}^p\}$ and proceeds to make VaR forecasts using the model $\mathcal{M}$. Inference within the model, including forecast analysis, depends crucially on the hypothesis that $q_\alpha \in \mathcal{M}$, i.e., if there exists some $\theta_0 \in \Theta$ such that $m_\alpha(W_{t-1}, \theta_0) = q_\alpha(W_{t-1})$ a.s. In parametric models the nuisance parameter $\theta_0$ belongs to $\Theta$, with $\Theta$ a compact set in a Euclidean space $\mathbb{R}^p$. Semiparametric and nonparametric specifications for $q_\alpha(\cdot)$ were also considered, see, e.g., Fan and Gu (2003), Martins-Filho and Yao (2006), and references therein, where $\theta_0$ belongs to an infinite-dimensional space. This article focuses on parametric VaR models where $\theta_0$ is finite-dimensional and can be estimated by a $\sqrt{R}$-consistent estimator, with $R$ denoting the (in-)sample size (cf. Assumption A4). Parametric VaR models are popular since the functional form $m_\alpha(W_{t-1}, \theta_0)$, jointly with the parameter $\theta_0$, describes in a very precise way the impact of the agent's information set on the VaR. The most popular parametric VaR models are those derived from traditional location-scale models such as autoregressive moving average (ARMA) and generalized autoregressive conditional heteroscedastic (GARCH) models. Our empirical analysis is focused on these models, although our theoretical results go beyond location-scale models. Alternative parametric VaR models can be found in Engle and Manganelli (2004), Koenker and Xiao (2006), and Gourieroux and Jasiak (2006), among many others.


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