A Unified Approach to Risk-Adjusted Performance

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Paul D. Kaplan, Ph.D., CFA
Vice President, Quantitative Research
Morningstar, Inc.
225 West Wacker Drive
Chicago, IL 60606

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1. Introduction

The most common measure of risk-adjusted performance for alpha generation strategies is the ratio of a portfolio's average active return to the standard deviation of active return, commonly known as the information ratio. The information ratio is appealing because it is easy to calculate, interpret, and apply.

Evaluating alternative strategies on the basis of the information ratio is valid so long as the probability distributions of alternative strategies differ primarily in their first two moments (mean and standard deviation). This is the case when all strategies being compared are well represented by the normal distribution. However, there can be important differences between strategies in the higher moments, namely skewness and kurtosis. For example, suppose that strategies A and B have the same information ratio, but that strategy A has one has a large amount of negative skewness and a high level of kurtosis (this is, has fat tails) and that strategy B is normally distributed. Although investors should find strategy A less desirable than strategy B, the information ratio fails to distinguish them. We need risk-adjusted performance measures that rank B superior to A.

A number of risk-adjusted performance measures have been developed to address the shortcomings of the information ratio when active return strategies are non-normal. These include the Sortino ratio, Omega, and the Stutzer index.[1] Kaplan and Knowles [2004] show that the Sortino ratio and Omega are special cases of a more general measure that they call Kappa.

The various risk-adjusted performance measures differ in theoretical motivation and mathematical form and can result in different rankings for non-normal distributions. However, they are more closely related to each other than is apparent. In this paper we unify all of these measures into a single family and expand on it. Our approach is based on expected utility theory with proportional risk aversion (PRA) with respect to active return. We say that a utility function exhibits PRA with respect to active return if the investor-specific risk aversion parameter multiplies active return in the utility function. In this way, the risk aversion parameter has the same effect on utility as the level of exposure to the alpha generator. We show that if each investor is free to set the level of exposure to the alpha generator, each risk-adjusted performance that we consider has a corresponding PRA utility function such that maximum expected utility is a monotonic transformation of the given measure.[2]

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[1] See Sortino [2001], Shadwick and Keating [2002], and Stutzer [2000] for descriptions of the Sortino ratio, Omega, and the Stutzer index respectively.

[2] In the case of Omega, there is no PRA utility function that generates the measure directly, but Omega is a limiting case for a family of PRA utility functions. See section 7.