MANAGEMENT SCIENCE
Vol. 37, No. 5, May 1991
Printed in U.S.A.

MEAN-ABSOLUTE DEVIATION PORTFOLIO OPTIMIZATION MODEL AND ITS APPLICATIONS TO TOKYO STOCK MARKET*

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The purpose of this paper is to demonstrate that a portfolio optimization model using the L_1 risk (mean absolute deviation risk) function can remove most of the difficulties associated with the classical Markowitz's model while maintaining its advantages over equilibrium models. In particular, the L_1 risk model leads to a linear program instead of a quadratic program, so that a large-scale optimization problem consisting of more than 1,000 stocks may be solved on a real time basis. Numerical experiments using the historical data of NIKKEI 225 stocks show that the L_1 risk model generates a portfolio quite similar to that of the Markowitz's model within a fraction of time required to solve the latter.

(PORTFOLIO OPTIMIZATION; L_1 RISK FUNCTION; LINEAR PROGRAMMING; MARKOWITZ'S MODEL; SINGLE-FACTOR MODEL)

1. Introduction

Markowitz's portfolio optimization model, contrary to its theoretical reputation, has not been used extensively in its original form to construct a large-scale portfolio. One of the most significant reasons behind this is the computational difficulty associated with solving a large-scale quadratic programming problem with a dense covariance matrix.

Several authors tried to alleviate this difficulty by using various approximation schemes (Sharpe 1967, 1971, Stone 1973) in the early years of the history. Also, the use of the index model enables one to reduce the amount of computation by introducing the notion of "factors" influencing the stock prices (Perold 1984, Sharpe 1963). Yet, these efforts are largely discounted because of the popularity of equilibrium models such as CAPM and APT which are computationally less demanding.

True that the idea of Markowitz is still being used indirectly because CAPM is based upon this model. But we must not forget that there is a fundamental difference between these two models. In particular, equilibrium models have to impose several unrealistic assumptions to derive a simple relation between the rate of return of individual assets and the market portfolio (see Elton and Gruber 1987, Sharpe 1964 for the details of CAPM). Unfortunately, however, the recent studies of data collected in Tokyo Stock Market show that this relation is very unstable. In fact, calculated "Beta" of stocks behave so erratically that the information provided by CAPM can at best serve as a first order approximation.

These observations motivated the authors' efforts to improve Markowitz's model both computationally and theoretically. In a series of papers (Konno 1988, 1989), we proposed a new portfolio optimization model using piecewise linear risk functions. In Konno (1989), we showed that our model can achieve the intention of Markowitz by solving a linear program instead of a "not so easy" quadratic program. Also, we demonstrated several nice properties of our model through preliminary numerical experiments using the historical data of 50 stocks included in OSAKA 50.

The main purpose of this paper is to demonstrate further that the L_1 risk model, a

* Accepted by W. T. Ziemba; received December 1989. This paper has been with the authors 2 months for 2 revisions.

0025-1909/91/3705/0519$01.25