

## Orderings and Probability Functionals Consistent with Preferences

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ABSTRACT This paper unifies the classical theory of stochastic dominance and investor preferences with the recent literature on risk measures applied to the choice problem faced by investors. First, we summarize the main stochastic dominance rules used in the finance literature. Then we discuss the connection with the theory of integral stochastic orders and we introduce orderings consistent with investors' preferences. Thus, we classify them, distinguishing several categories of orderings associated with different classes of investors. Finally, we show how we can use risk measures and orderings consistent with some preferences to determine the investors' optimal choices.

KEY WORDS: Stochastic dominance, probability functionals, integral orderings, coherent and convex measures, utility theory, efficient choices

JEL Classification: G11, C44, C61

## 1. Introduction

In this paper, we classify risk/uncertainty orderings and measures consistent with investors' preferences. In particular, we present a general and unifying framework of the theory of orderings by examining the connection with recent studies on risk measures.

Utility theory classifies the optimal choices for different categories of market agents (for example, risk-averse, non-satiable, non-satiable risk averse) under ideal market conditions. Roughly speaking, in utility theory the ordering of uncertain choices begins with the selection of a finite number of axioms that characterize the preferences for a given class of market agents. The second step of the theory involves representing the preferences of market agents using 'utility functionals' that summarize the decision makers' behaviour. Consequently, using the correspondence between the orderings of utility functionals and the orderings of random variables,

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