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RISK AVERSION IN THE SMALL AND IN THE LARGE[1]

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This paper concerns utility functions for money. A measure of risk aversion in the small, the risk premium or insurance premium for an arbitrary risk, and a natural concept of decreasing risk aversion are discussed and related to one another. Risks are also considered as a proportion of total assets.

1. SUMMARY AND INTRODUCTION

LET $u(x)$ BE a utility function for money. The function $r(x)=-u''(x)/u'(x)$ will be interpreted in various ways as a measure of local risk aversion (risk aversion in the small); neither $u''(x)$ nor the curvature of the graph of u is an appropriate measure. No simple measure of risk aversion in the large will be introduced. Global risks will, however, be considered, and it will be shown that one decision maker has greater local risk aversion $r(x)$ than another at all x if and only if he is globally more risk-averse in the sense that, for every risk, his cash equivalent (the amount for which he would exchange the risk) is smaller than for the other decision maker. Equivalently, his risk premium (expected monetary value minus cash equivalent) is always larger, and he would be willing to pay more for insurance in any situation. From this it will be shown that a decision maker's local risk aversion $r(x)$ is a decreasing function of x if and only if, for every risk, his cash equivalent is larger the larger his assets, and his risk premium and what he would be willing to pay for insurance are smaller. This condition, which many decision makers would subscribe to, involves the third derivative of u, as $r'\leqq 0$ is equivalent to $u'''u'\geqq u''^2$. It is not satisfied by quadratic utilities in any region. All this means that some natural ways of thinking casually about utility functions may be misleading. Except for one family, convenient utility functions for which $r(x)$ is decreasing are not so very easy to find. Help in this regard is given by some theorems showing that certain combinations of utility functions, in particular linear combinations with positive weights, have decreasing $r(x)$ if all the functions in the combination have decreasing $r(x)$.

The related function $r^*(x)=xr(x)$ will be interpreted as a local measure of aversion to risks measured as a proportion of assets, and monotonicity of $r^*(x)$ will be proved to be equivalent to monotonicity of every risk's cash equivalent measured as a proportion of assets, and similarly for the risk premium and insurance.

These results have both descriptive and normative implications. Utility functions for which $r(x)$ is decreasing are logical candidates to use when trying to describe the behavior of people who, one feels, might generally pay less for insurance against

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