

# A Jigsaw Puzzle of Basic Risk-adjusted Performance Measures

*In 1997, Modigliani and Modigliani developed the risk-adjusted performance measure RAP (often called M-squared), which is now widely accepted in theory and practice. Their measure has further increased investor awareness of risk-adjusted performance measurement. However, this measure uses the standard deviation as the relevant measure of risk, and, therefore, is relevant only to investors who invest their entire savings into a single fund. In this article the authors present a jigsaw puzzle of basic risk-adjusted performance measures, which helps to better understand the key links between these measures. In doing so they include a hardly known measure: the market risk-adjusted performance (MRAP). While closely related to the Modigliani measure, the MRAP measures returns relative to market risk instead of total risk. Thus, the MRAP is suitable for investors who invest in many different assets.*

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Modigliani and Modigliani (1997) introduced “risk-adjusted performance” (RAP) as a new measure to gauge the performance of investment funds. It uses the standard deviation as the relevant measure of risk and can be easily understood by investors. By now, the RAP (often also referred to as “M-squared” or “M<sup>2</sup>”) has been widely accepted in theory and practice (see, for example, Sharpe, Alexander and Bailey (1999), Hopkins and Acton (1999)) and serves as a basis for further performance measures.<sup>2</sup> However, the RAP is based on total risk, taking only the fund’s standard deviation of returns into account. Thus, in a strict sense, it only suits investors who exclusively invest in a portfolio consisting of a single investment fund and either investing or borrowing at the risk-free rate.<sup>3</sup>

We introduce a jigsaw puzzle or system of basic risk-adjusted performance measures that helps understand the key differences between these performance measures and clarifies the links between them (see Table 1). In doing so we include a still hardly known measure: the “market risk-adjusted performance” (MRAP). This performance measure is similar to RAP, except that it is based on market risk instead of total risk. Since the MRAP considers the market risk of the funds, it is also

relevant for investors who invest in a variety of assets. This permits a sensible comparison of the performance of investment funds on the basis of a measure that can easily be interpreted.

In order to explain the key differences between the basic performance measures and to point out the links between them, we present an example. It is based on fictitious annualized returns of a market index and of two funds A and B (see Table 2).<sup>4</sup> Investors can invest or borrow at the constant risk-free rate of 3% p.a. at any time.

## Definitions and Notation

Before proceeding to operational formulae, we summarize the notation to be used as:

$r_f$  = risk-free interest rate

$\mu_i$  = average return of fund i

$\mu_M$  = average return of the market index

$\sigma_i$  = standard deviation of the returns of fund i

$\sigma_M$  = standard deviation of the returns of