

Risk, VaR, CVaR and their associated Portfolio Optimizations when Asset Returns have a Multivariate Student T Distribution

William T. Shaw*

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Abstract

We show how to reduce the problem of computing VaR and CVaR with Student T return distributions to evaluation of analytical functions of the moments. This allows an analysis of the risk properties of systems to be carefully attributed between choices of risk function (e.g. VaR vs CVaR); choice of return distribution (power law tail vs Gaussian) and choice of event frequency, for risk assessment. We exploit this to provide a simple method for portfolio optimization when the asset returns follow a standard multivariate T distribution. This may be used as a semi-analytical verification tool for more general optimizers, and for practical assessment of the impact of fat tails on asset allocation for shorter time horizons.

Keywords: VaR, CVaR, Portfolio Optimization, VaR Optimization, CVaR Optimization, Optimisation.

1 Introduction

The non-Gaussian nature of asset returns has been established for many years from many perspectives. From the point of view of statistical moment studies, Fama [3] and Mandelbrot [7] demonstrated the excess kurtosis of asset returns in the 1960s. Studies published in 2003 [6] focused on tail analysis, claiming that the tails of return distribution functions exhibit power-law behaviour, often with inverse *cubic* decay. Maximum Likelihood analysis has also been pursued. Fergusson and Platen [4] exhibited the importance of Student-T characteristics in *daily log-returns* of major indices. In the follow studies we will use the Student T as a *model* of asset returns, in that it can simultaneously reproduce excess kurtosis, power-law tails and be at or close to the MLE distributional estimate in a variety of situations¹. This is not to argue that this distribution is a universal panacea. Rather, it serves three other purposes. First, its manifestly greater likelihood in matching observed returns suggest that it is useful model for exploring risk characteristics and asset allocation in its own right. Second, it serves as a useful semi-analytical benchmark for more general computational models of optimization, to ensure their quality on an otherwise hard-to-obtain problem. VaR/CVaR optimization has been pioneered by Rockafeller and Uryasev [11], and general risk measure optimization by Shaw [13]. Finally, the reduction of VaR and CVaR optimization to an essentially closed-form problem description, with an objective function that may be explicitly

*Department of Mathematics and Computer Science, University College London. (w.shaw@ucl.ac.uk).

¹More recent work by Platen and co-workers [2] finds the T or Variance Gamma as an MLE amongst the hyperbolic distributional family. My own preliminary studies also find Johnson-SU distributions as a contender, with all 3 types many orders of magnitude more likely than Gaussian on equity index and asset log-returns