
A Portfolio Performance Index

Michael Stutzer

Fund managers may sensibly be averse to earning a time-averaged portfolio return that is less than the average return of some designated benchmark. When a portfolio is expected to earn a higher average return than the benchmark return, the probability that it will not approaches zero asymptotically at a computable exponential decay rate. The probability decay rate is thus proposed here as a new portfolio "performance index." In the widely analyzed special case in which returns are normally distributed, the new performance-index-maximizing portfolio is the same as the popular Sharpe-ratio-maximizing portfolio. The results of the two approaches generally differ, however, because of nonnormal levels of skewness and/or kurtosis in the portfolio attributable to large asymmetrical economic shocks or investments in options and other derivative securities. An illustrative example will show that the new index is easy to implement and, consistent with empirical evidence on portfolio choice, favors investments with positively skewed returns.

Pension and endowment portfolio analysts need a meaningful yet practical way to rank-order feasible portfolios. Perhaps the most widely used measure for this purpose is the Sharpe ratio. The Sharpe ratio of a portfolio is its expected excess return relative to a chosen benchmark (usually the "riskless" rate of interest) divided by the standard deviation of its excess return. When returns are individually and identically normally distributed, risk-averse investors will choose a portfolio that is mean-standard deviation efficient, and Sharpe (1994) summarized the foundational case for using the Sharpe ratio to evaluate (*ex ante*) the growth (i.e., mean) versus security (i.e., standard deviation) trade-off in these portfolios. In practice (*ex post*), a historical time series of portfolio returns minus benchmark returns is used to calculate historical average excess return and historical standard deviation as estimates of the unknown *expected* excess return and standard deviation. And in practice, portfolios are ranked by the size of this *ex post* Sharpe measure.

But what is to be done when the returns are not normally distributed? The theoretical foundation for the Sharpe measure does not apply when excess returns deviate from the normal because of large absolute values of skewness and/or kurtosis.

Michael Stutzer is professor of finance at the University of Iowa.

Such nonnormalities in a portfolio may arise from large asymmetrical economic shocks, investments in options and other derivative securities with inherently asymmetrical returns, limited liability (bankruptcy) effects on asset returns, or other causes.¹ Moreover, a suggestion that has been made for a long time now is that investors value positive skewness of returns (e.g., Kraus and Litzenberger 1976). The Sharpe measure does not consider skewness at all.

Alternatives to the Sharpe Ratio

An alternative performance index should satisfy the following desiderata:

- The index should rank-order portfolios in accord with the Sharpe ratio when returns are normally distributed.
- When returns are not normally distributed, the index should reflect skewness preference while retaining the Sharpe ratio's useful statistical interpretation and ease of implementation.
- The index should be derived from a sound behavioral foundation that is free of unspecified and unknowable parameters and is relevant to many fund managers.

Three general approaches have been used to construct alternative performance indexes: *ad hoc* modifications to the Sharpe ratio, expected utility functions, and expected return relationships implied by perfect financial market equilibrium resulting from investors who maximize expected