A Minimax Portfolio Selection Rule with Linear Programming Solution

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A new principle for choosing portfolios based on historical returns data is introduced; the optimal portfolio based on this principle is the solution to a simple linear programming problem. This principle uses minimum return rather than variance as a measure of risk. In particular, the portfolio is chosen that minimizes the maximum loss over all past observation periods, for a given level of return. This objective function avoids the logical problems of a quadratic (nonmonotone) utility function implied by mean-variance portfolio selection rules. The resulting minimax portfolios are diversified; for normal returns data, the portfolios are nearly equivalent to those chosen by a mean-variance rule. Framing the portfolio selection process as a linear optimization problem also makes it feasible to constrain certain decision variables to be integer, or 0–1, valued; this feature facilitates the use of more complex decision-making models, including models with fixed transaction charges and models with Boolean-type constraints on allocations.
(*Mean-Variance Analysis; Optimization; Utility Theory; Volatility*)

1. Introduction

Sharpe (1971) has remarked that "if the essence of the portfolio analysis problem could be adequately captured in a form suitable for linear programming methods, the prospect for practical application would be greatly enhanced." In this paper, a portfolio selection principle is introduced, which is such that the optimal portfolio is a solution to a simple linear program. The principle is referred to as "minimax": The optimal portfolio is defined as that one that would minimize the maximum loss (in dollars) over all past historical periods, subject to a restriction on the minimum acceptable average return across all observed time periods. This principle leads to portfolio selections similar to those obtained by the mean-variance selection rule of Markowitz (1991), in the case when returns have a sample distribution that is approximately multivariate normal.

Since linear programming is becoming a standard feature on personal computer spreadsheet programs, this new minimax rule has the potential to make portfolio optimization a tool accessible to any financial manager. Framing the portfolio selection process as a linear optimization problem also makes it feasible to constrain certain decision variables to be integer, or 0–1, valued; this feature facilitates the use of more complex decision-making models. For example, a linear-integer programming model can accommodate fixed transaction charges, a cost commonly encountered by portfolio managers. In addition to the minimax rule's computational convenience, the method may also have logical advantages when the returns are nonnormally distributed, and when the investor has a strong form of risk aversion.

1.1. The Minimax Portfolio Selection Rule

Suppose data are observed for N securities, over T time periods. Let

y_{jt} = Return on one dollar invested in security j in time period t.

$\bar{y}_j$ = Average Return on security j $= 1/T \sum_{t=1}^{T} y_{jt}$.

w_j = Portfolio allocation to security j.

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